

V 11.05.2018.

Snižavajući broj rješenja lin. D.J.-a kada su

poznata parcijskarna rješenja /

$$X' = A(t) \cdot X \quad - \text{sistem reda } n$$

e_1, \dots, e_m - nezavisna rješenja učin

$$\Phi_1 = \begin{pmatrix} e_{11} & \dots & e_{1m} \\ \vdots & \ddots & \vdots \\ e_{n1} & \dots & e_{nm} \end{pmatrix}_{n \times m}$$

$$\Phi_2 = \left(\begin{array}{c|c} 0 & \} m \\ \hline E & \} n-m \\ \hline n-m & n \times (n-m) \end{array} \right)$$

$$\Phi = [\Phi_1, \Phi_2]_{n \times n}$$

$$X = \Phi Y$$

$$X' = \Phi' Y + \Phi Y'$$

$$\Phi' Y + \Phi Y' = A \Phi Y$$

$$\Phi Y' = (A \Phi - \Phi') Y$$

$$A \Phi = A [\Phi_1, \Phi_2]$$

$$= [A \Phi_1, A \Phi_2]$$

$$\Phi' = [\Phi_1', 0] \quad (\Phi_2 \text{ je const. matrica})$$

$$A \Phi - \Phi' = \underbrace{[A \Phi_1 - \Phi_1', 0]}_{0}, A \Phi_2 =$$

$$= [0, A \Phi_2]$$

$$\Phi Y' = [0, A \Phi_2] \cdot Y$$

$$Y' = [0, \Phi^{-1} A \Phi_2] Y \xrightarrow{n \times (n-m)}$$

$$m \times \begin{pmatrix} Y_1' \\ Y_2' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \xrightarrow{n \times n \times n-m}$$

$$Y_1' = Q_1 Y_2$$

$$Y_2' = Q_2 Y_2$$

Nova ječenja preko $X = \Phi Y$.

$$\textcircled{1} \quad x_1^1 = 3x_1 - 2x_2 + x_3$$

$$x_2^1 = x_1 + x_2 - 3x_3$$

$$x_3^1 = x_2 - 2x_3$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Treba da nademo još dva rešenja.

$$\phi_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad m=1$$

$$\phi_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

n-m

$$\phi = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 1 & -3 \\ 0 & 1 & -2 \end{pmatrix}$$

$$\phi^{-1} A \phi_2 = ?$$

$$A \phi_2 = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 1 & -3 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -3 \\ 1 & -2 \end{pmatrix}$$

$$\phi^{-1} = ?$$

$$\det \phi = 1$$

$$\phi^{-1} = \begin{pmatrix} 1 & -2 & -1 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\phi^{-1} A \phi_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & -3 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 5 & -5 \\ 3 & -3 \end{pmatrix}$$

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \left(\begin{array}{ccc|c} 0 & -2 & 1 & Y_1 \\ 0 & 5 & -5 & \\ 0 & 3 & -3 & \end{array} \right) \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

$\rightarrow \text{det} A = 1$
 $\rightarrow \text{det } n-m = 3-1 = 2$

$$Y_1 = (-2 \ 1) Y_2$$

$$Y_2 = \begin{pmatrix} 5 & -5 \\ 3 & -3 \end{pmatrix} Y_2$$

$$\det (A_2 - \lambda E) = (5-\lambda)(-3-\lambda) + 15 = \dots = \lambda^2 - 2\lambda$$

$$\lambda = 0 \Rightarrow f_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad Y_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot e^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda = 2 \Rightarrow f_2 = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad Y_2 = \begin{pmatrix} 5 \cdot e^{2t} \\ 3 \cdot e^{2t} \end{pmatrix}$$

Y_2

$$Y_1 = (-2 \ 1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1$$

$$\Rightarrow Y_1 = -t$$

$$Y = \begin{pmatrix} -t \\ 1 \\ 1 \end{pmatrix}$$

$$Y_1 = (-2 \ 1) \begin{pmatrix} 5 e^{2t} \\ 3 e^{2t} \end{pmatrix} = -7 e^{2t}$$

$$\Rightarrow Y_1 = -\frac{7}{2} e^{2t}$$

$$Y = \begin{pmatrix} -\frac{7}{2} e^{2t} \\ 5 e^{2t} \\ 3 e^{2t} \end{pmatrix}$$

$$e_2 = \phi \cdot Y = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -t \\ 1 \\ 1 \end{pmatrix} = \dots$$

$$e_3 = \phi \cdot Y = \begin{pmatrix} -\frac{7}{2} e^{2t} \\ 5 e^{2t} \\ 3 e^{2t} \end{pmatrix} = \dots$$

$$X = c_1 e_1 + c_2 e_2 + c_3 e_3 = \dots$$

$$\textcircled{2} \text{ und } \begin{aligned} x_1' &= tx_1 + x_2 - tx_3 \\ x_2' &= x_1 - x_3 \\ x_3' &= \frac{1}{t}x_1 + x_2 - \frac{1}{t}x_3 \end{aligned}$$

$$\varphi_1 = \begin{pmatrix} t \\ 1 \\ t \end{pmatrix}$$

$$\textcircled{3} \quad \phi = \begin{pmatrix} t & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{t} & 0 & 1 \end{pmatrix} \quad A = \begin{pmatrix} t & 1 & -t \\ 1 & 0 & -1 \\ \frac{1}{t} & 1 & -\frac{1}{t} \end{pmatrix}$$

$$\det \phi = t.$$

$$\phi^{-1} = \frac{1}{t} \begin{pmatrix} 1 & 0 & 0 \\ -1 & t & 0 \\ -t & 0 & t \end{pmatrix} = \begin{pmatrix} \frac{1}{t} & 0 & 0 \\ -\frac{1}{t} & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\phi^{-1} A = \begin{pmatrix} 1 & \frac{1}{t} & -1 \\ 0 & -\frac{1}{t} & 0 \\ \frac{1-t^2}{t} & 0 & \frac{t^2-1}{t} \end{pmatrix}$$

$$\phi^{-1} A \varphi_2 = \begin{pmatrix} \frac{1}{t} & -1 \\ -\frac{1}{t} & 0 \\ 0 & t-\frac{1}{t} \end{pmatrix}$$

$$1. \left\{ \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} \right. = \left. \begin{pmatrix} 0 & \frac{1}{t} & -1 \\ 0 & -\frac{1}{t} & 0 \\ 0 & 0 & t-\frac{1}{t} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\} \quad y_2 = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$y_1' = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$y_2' = \begin{pmatrix} -1/t & 0 \\ 0 & t-1/t \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \Rightarrow z_1' = -\frac{1}{t} z_2 \Rightarrow z_1 = \frac{c_1}{t} \\ z_2' = (t-\frac{1}{t}) z_2 \Rightarrow z_2 = \frac{c_2 e^{\frac{t^2-1}{t}}}{t}$$

$$\Rightarrow y_2 = \begin{pmatrix} 1/t \\ 0 \end{pmatrix} \text{ rli } y_2 = \begin{pmatrix} 0 \\ \frac{e^{\frac{t^2-1}{t}}}{t} \end{pmatrix}$$

$$y_1' = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{t^2} \Rightarrow y_1 = -\frac{1}{t} + c_3$$

$$Y = \begin{pmatrix} -\frac{1}{t} \\ \frac{1}{t} \\ 0 \end{pmatrix}$$

$$\varphi_2 = \underbrace{\begin{pmatrix} t & 0 & 0 \\ 1 & 1 & 0 \\ t & 0 & 1 \end{pmatrix}}_{\phi} \begin{pmatrix} -\frac{1}{t} \\ \frac{1}{t} \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Konstanta $\varphi_2 = \begin{pmatrix} 0 \\ e^{\frac{t}{2}} \\ t \end{pmatrix}$ odrediti φ_1, p_a

i onda $\varphi_3 = \phi \cdot Y$

$\varphi_1, \varphi_2, \varphi_3$ sve bazu.

Za φ_1 :

$$\cancel{x} \quad x' = Ax, \quad A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \\ -1 & 0 & -2 \end{pmatrix} \quad \varphi_1 = \begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix}$$

(Autonam)

Dinamički sistemi

$$\begin{cases} x_1' = f_1(x_1, \dots, x_n) \\ \vdots \\ x_n' = f_n(x_1, \dots, x_n) \end{cases}$$

dinamički sistem

integralna kriva
 $X = X(t)$
 $t \in \mathbb{R}$

Teorema: Fazne trajektorije dinamičkog sistema mogu biti tačka, glatka kriva bez samopresjeka, zatvorena glatka kriva.

$$X' = F(X)$$

$$X_0 \in D$$

potraži ravnoteže

$$F(X_0) = 0$$

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 \\ x_2' = a_{21}x_1 + a_{22}x_2 \end{cases}$$

$$\text{Teorema: } x_1' = f_1(x_1, x_2)$$

$$x_2' = f_2(x_1, x_2)$$

Tada su fazne trajektorije ovog sistema integralne krive dif.

$$\text{j-ne } f_2(x_1, x_2)dx_1 - f_1(x_1, x_2)dx_2 = 0$$

$\lambda_1 \neq \lambda_2$ (sv.vr. matrice A)

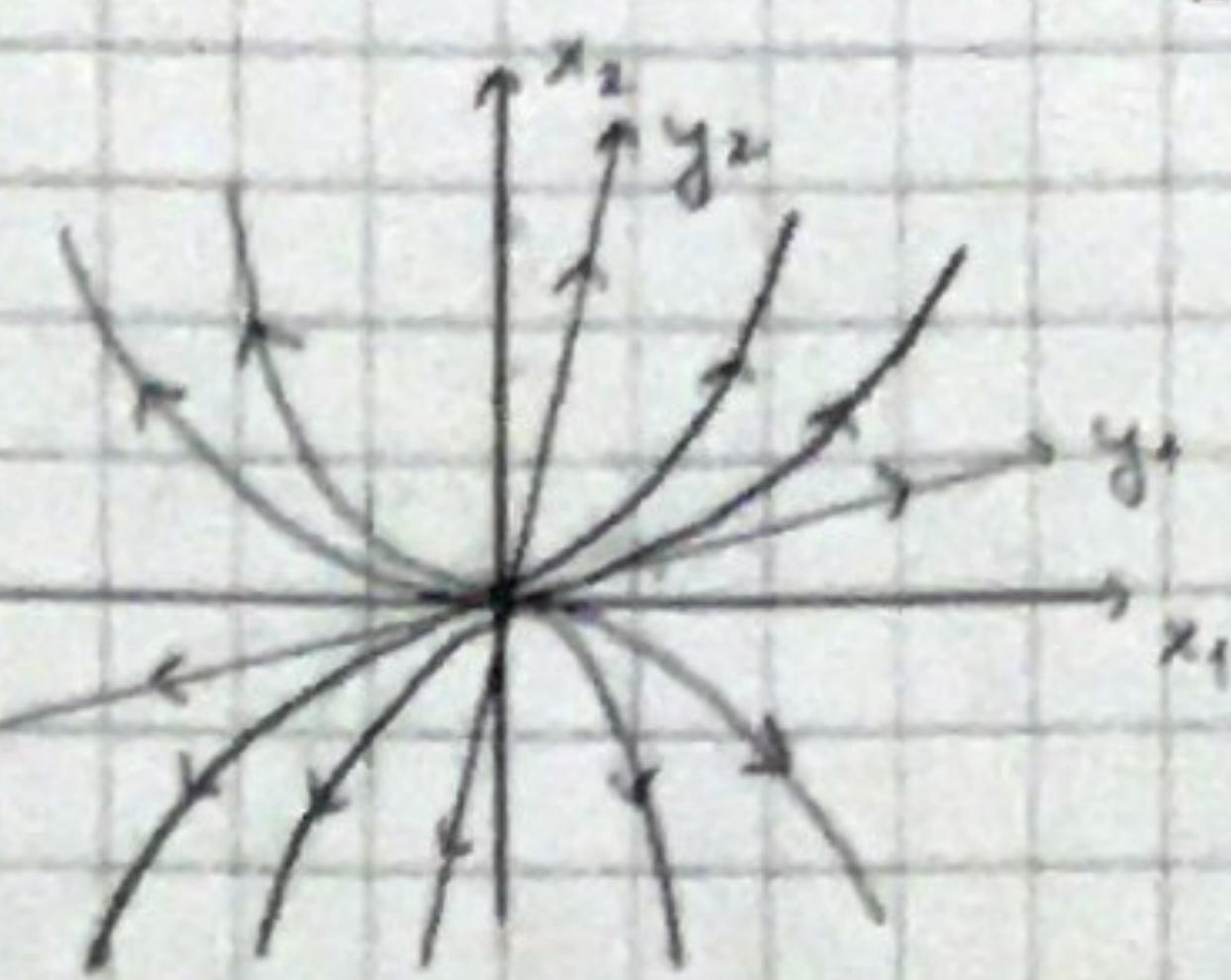
$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$\lambda_1 \neq \lambda_2$

nestabilni

čvor

$0 < \lambda_1 < \lambda_2$



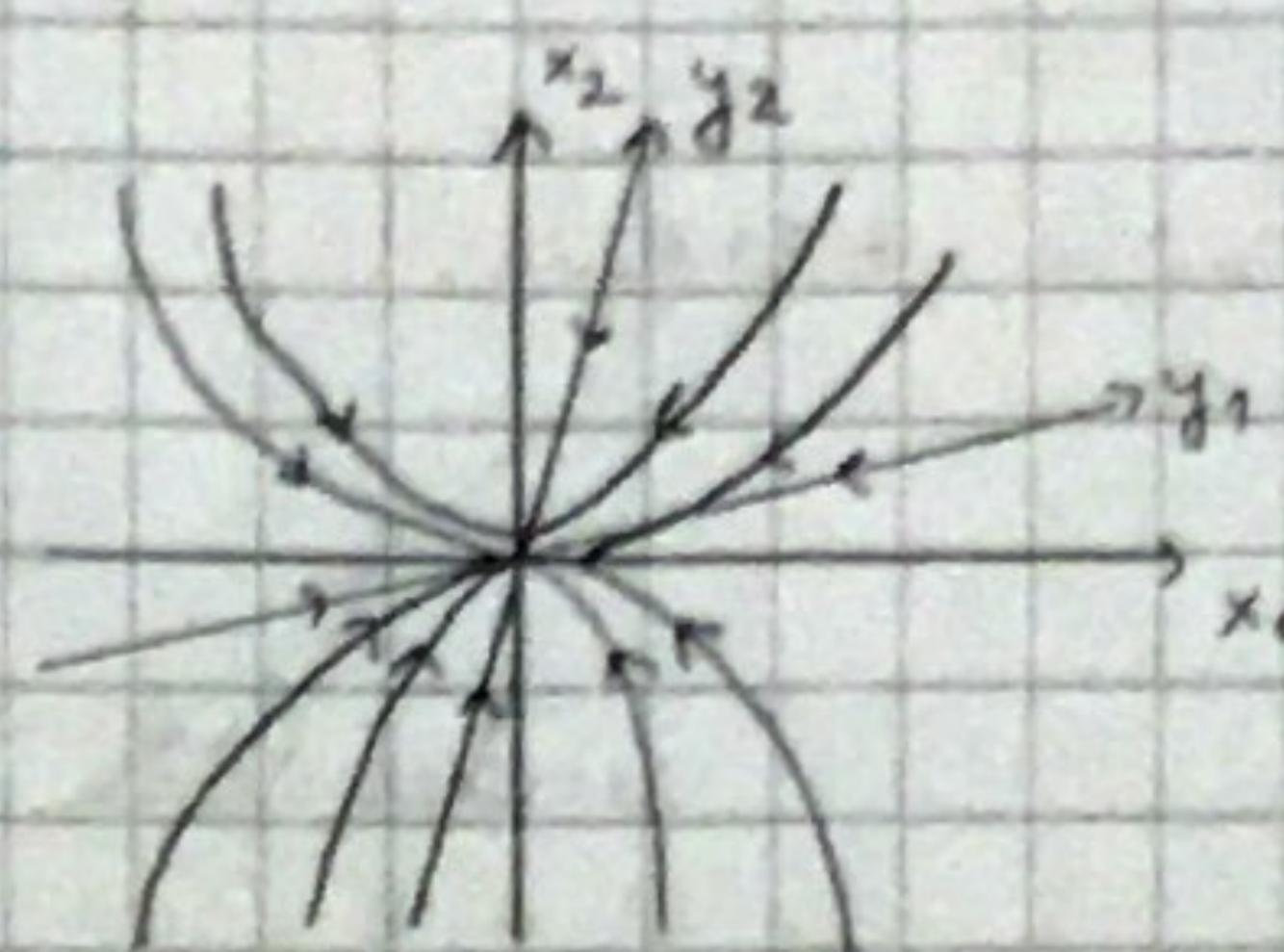
trajektorije se udaljavaju
od nule

$\lambda_1 \neq \lambda_2$ neg.

stabilan

čvor

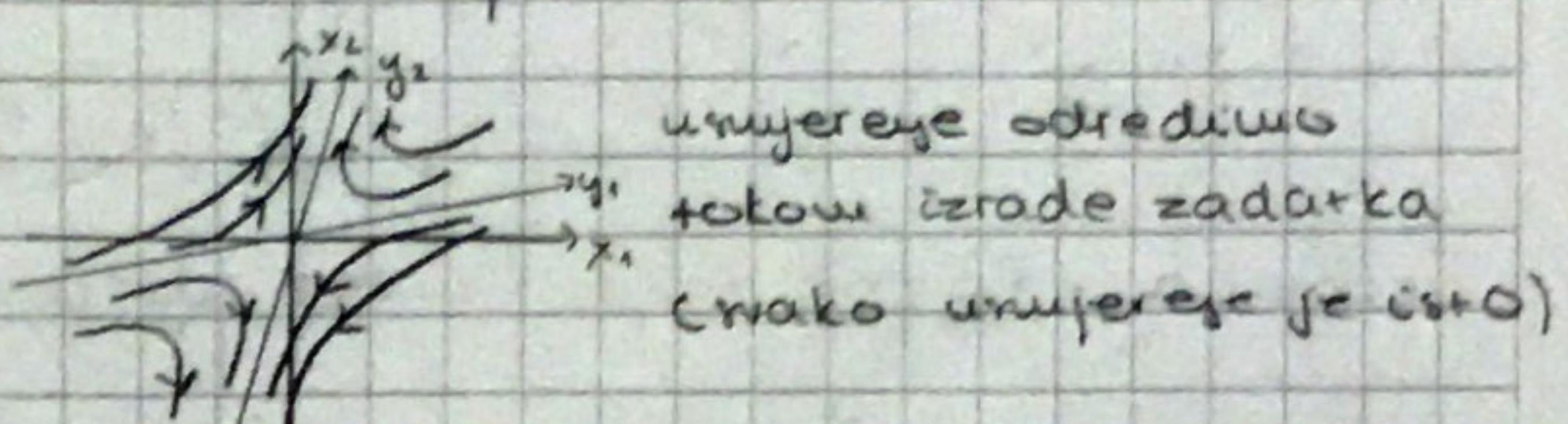
$0 > \lambda_1 > \lambda_2$



približavaju se nuli

$\lambda_1 < 0 < \lambda_2$

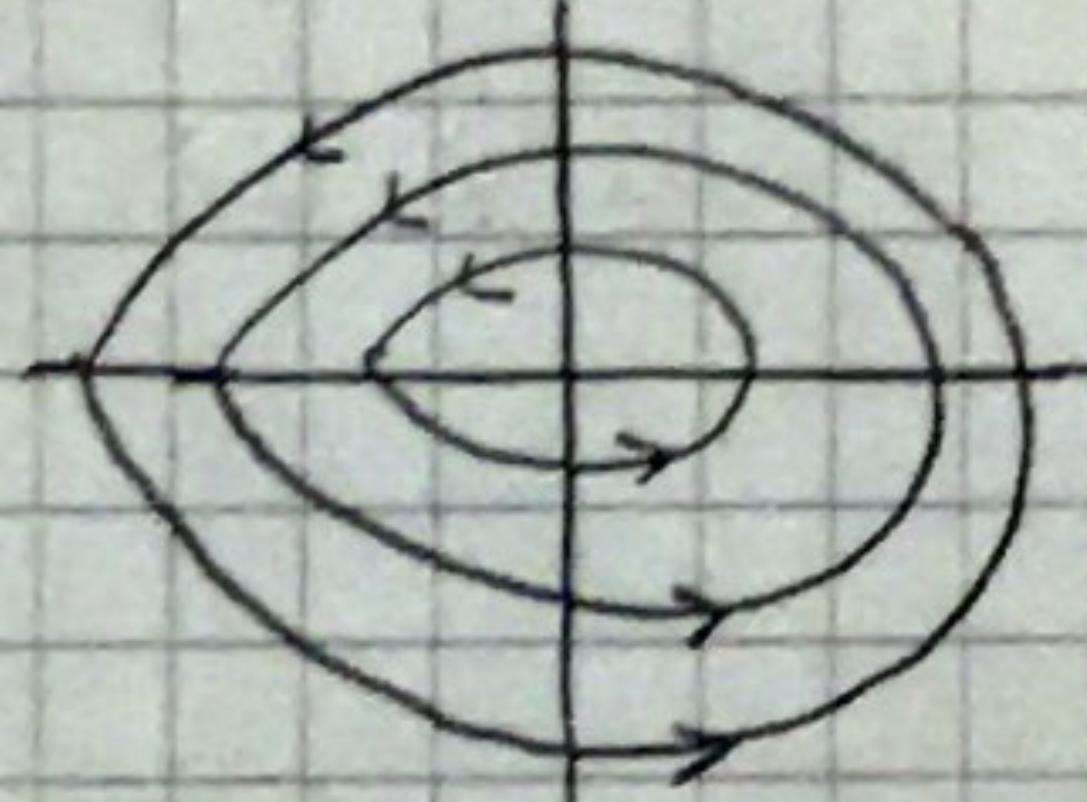
sedlo



uničevaju odredinu
takow izrade zadatka
(nako uničevje je čit=0)

$\lambda = \pm i\beta$

centar



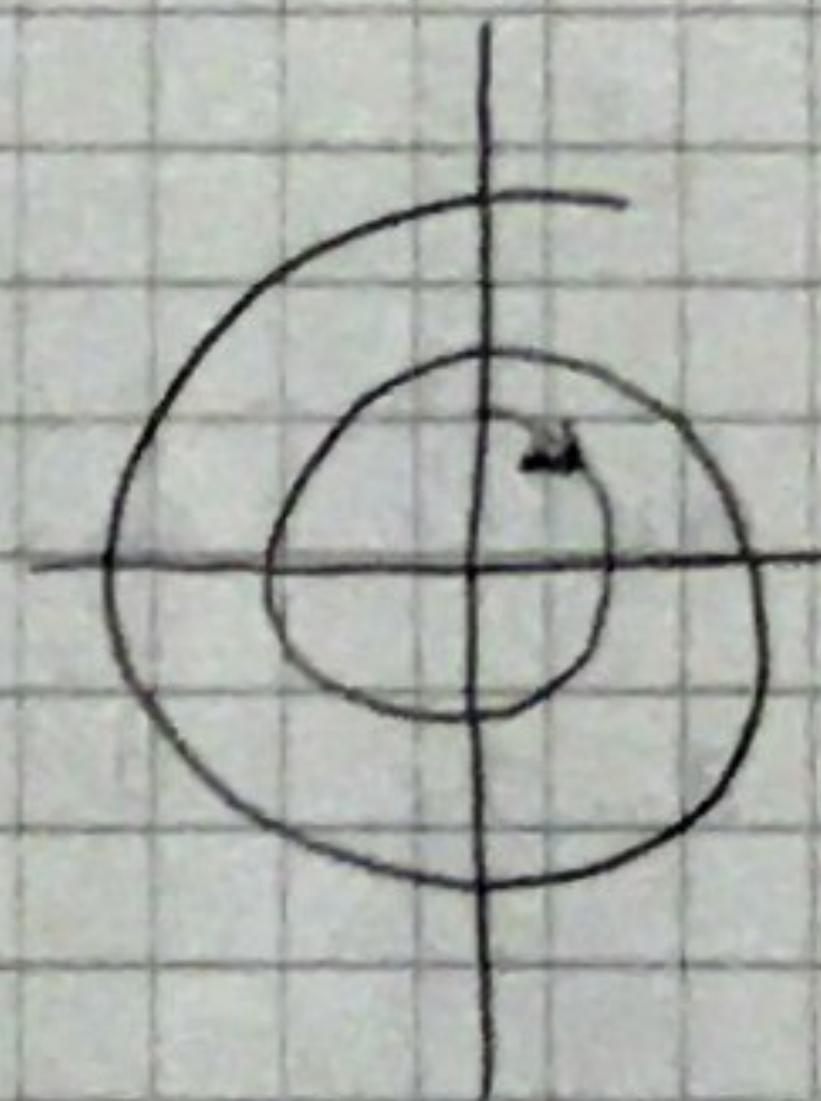
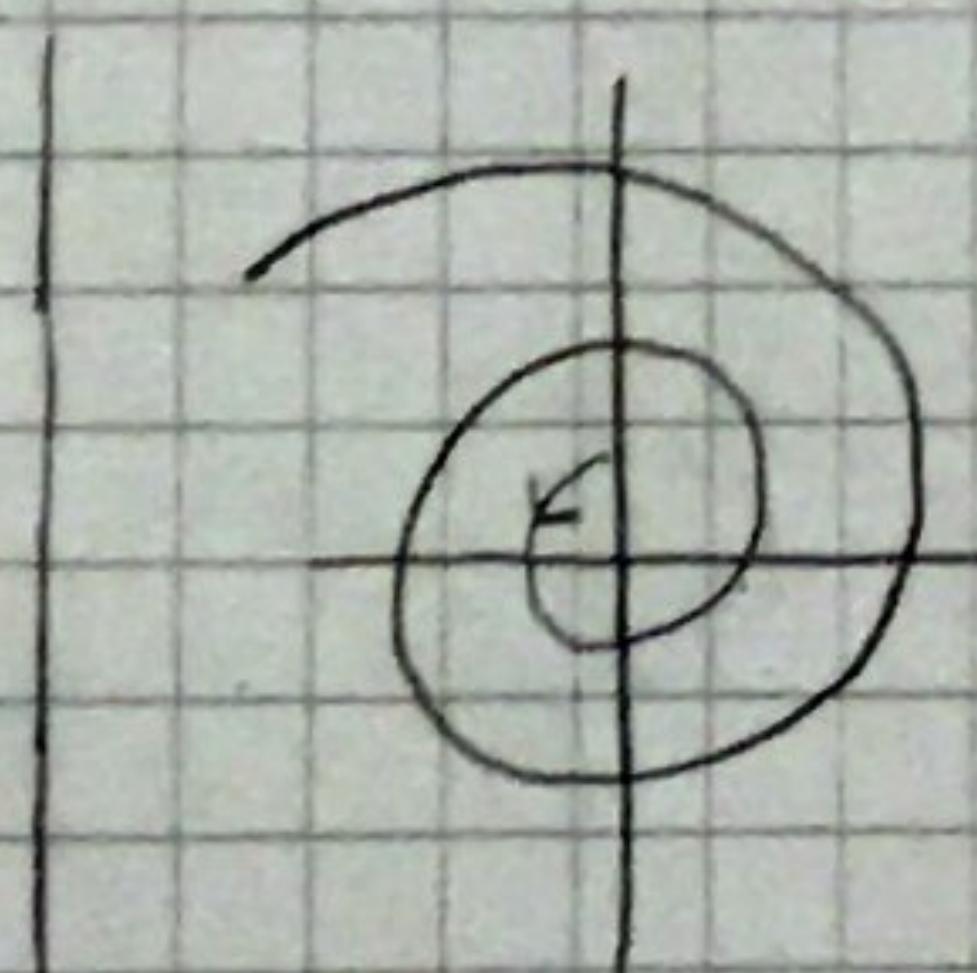
DAE MODULES

$\lambda_{1,2} = \alpha \pm i\beta$

nestabilan

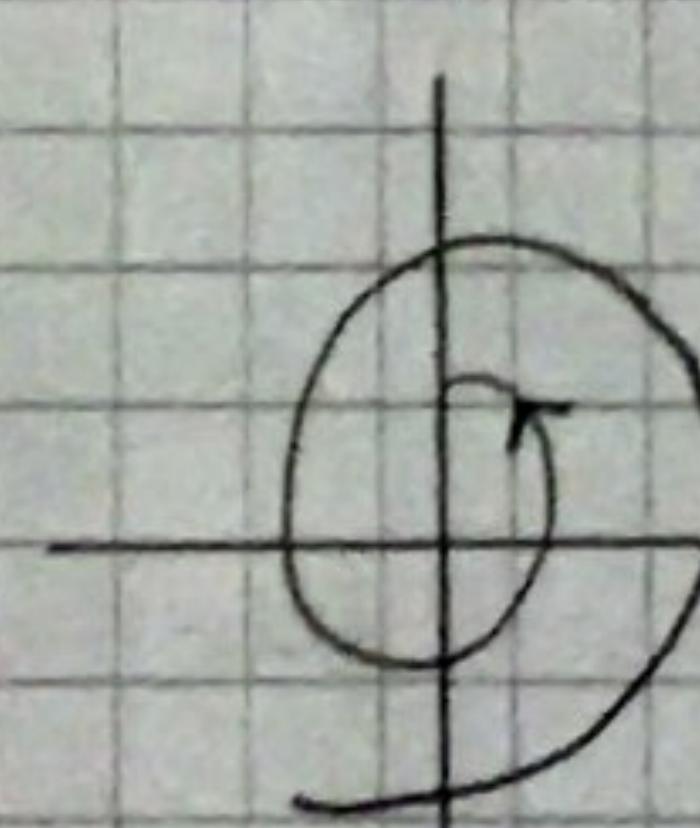
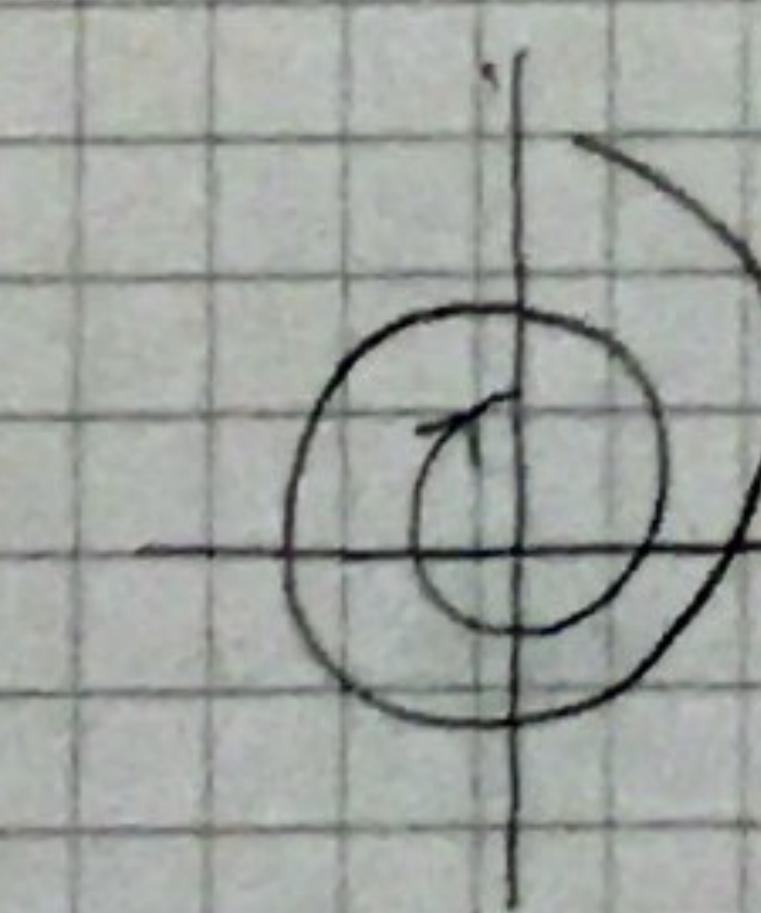
$\alpha > 0$

fokus



$\alpha < 0$

stabilan fokus



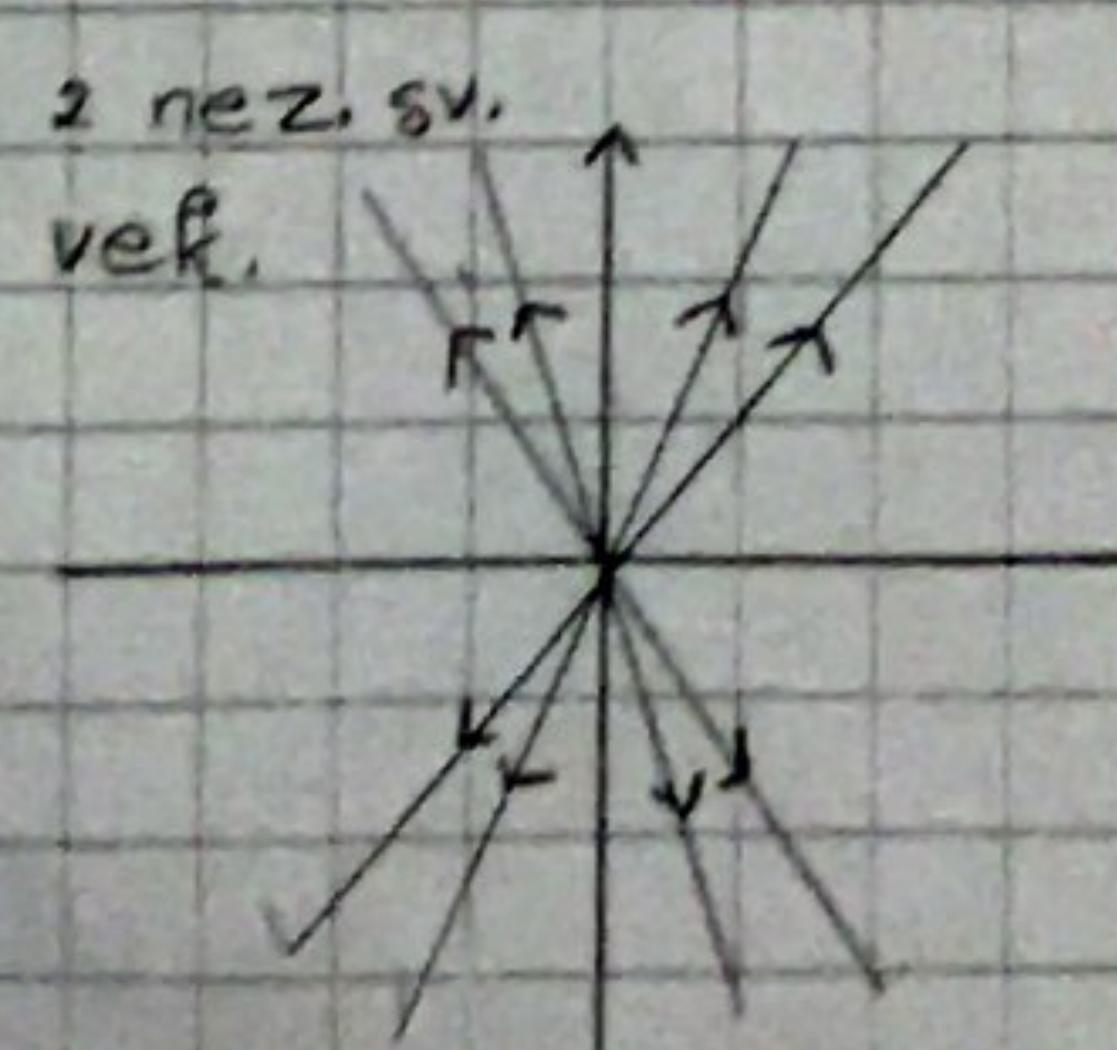
$\lambda_1 = \lambda_2$

nestabilni

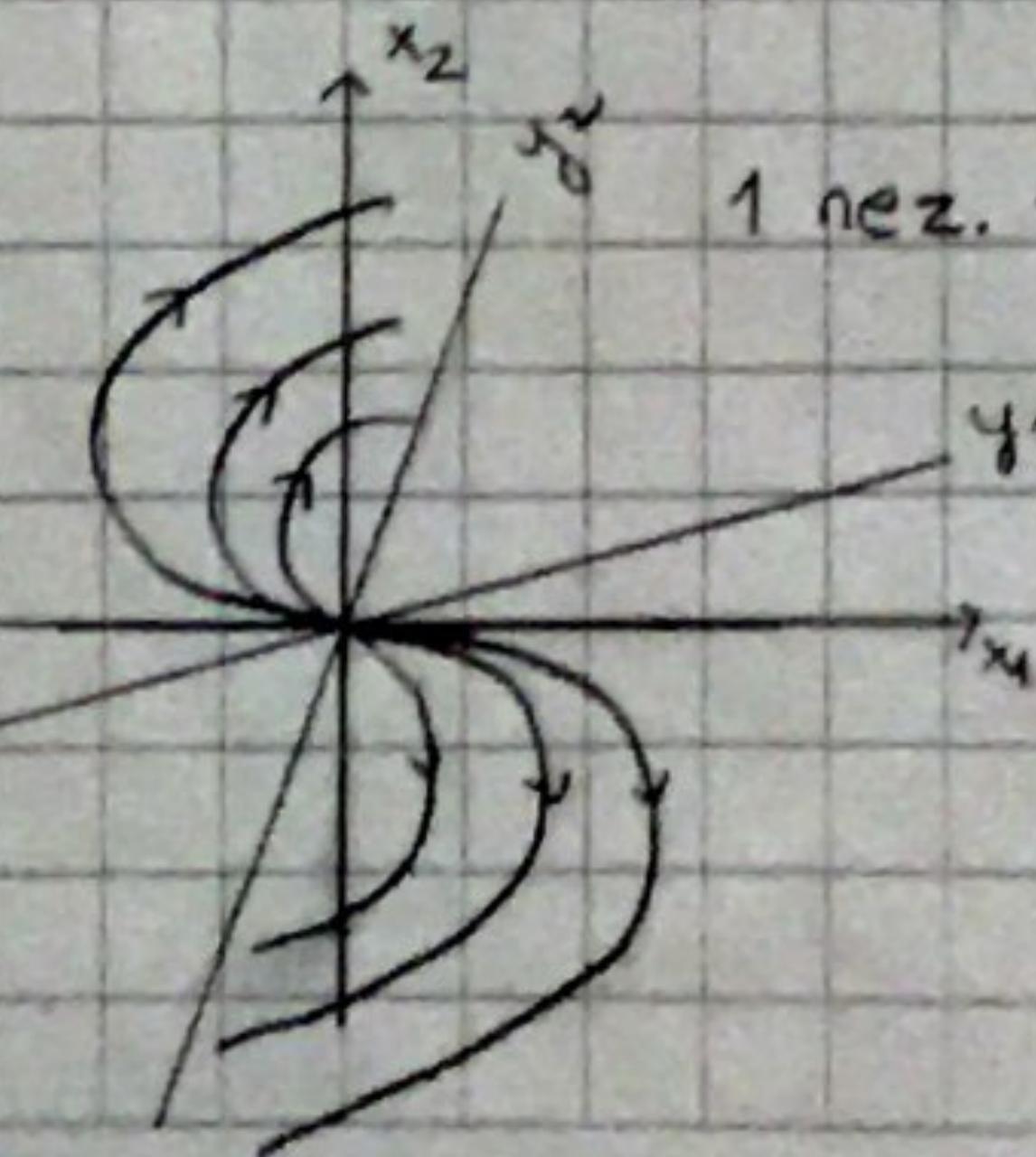
dekritični

$\lambda_1 > 0$

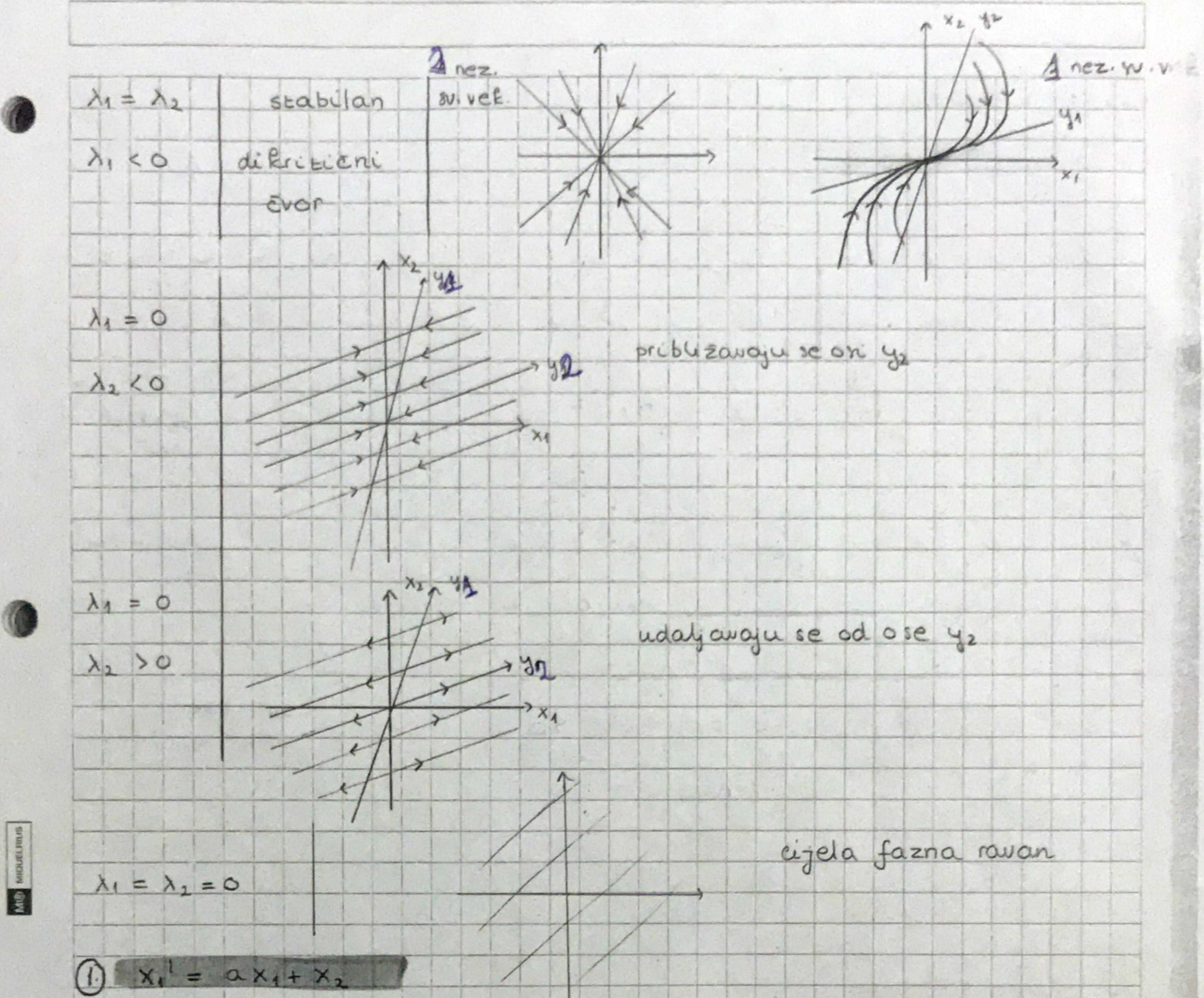
čvor



2 nez. sv.
vek.



1 nez. sv. vek.



$$① \quad x_1' = ax_1 + x_2$$

$$x_2' = -x_1 + ax_2$$

$$A = \begin{pmatrix} a & 1 \\ -1 & a \end{pmatrix}$$

$$\det(A - \lambda E) = (\lambda - a)^2 + 1 \Rightarrow \dots \lambda_1, \lambda_2 = a \pm i$$

1° $a = 0 \Rightarrow$ centar

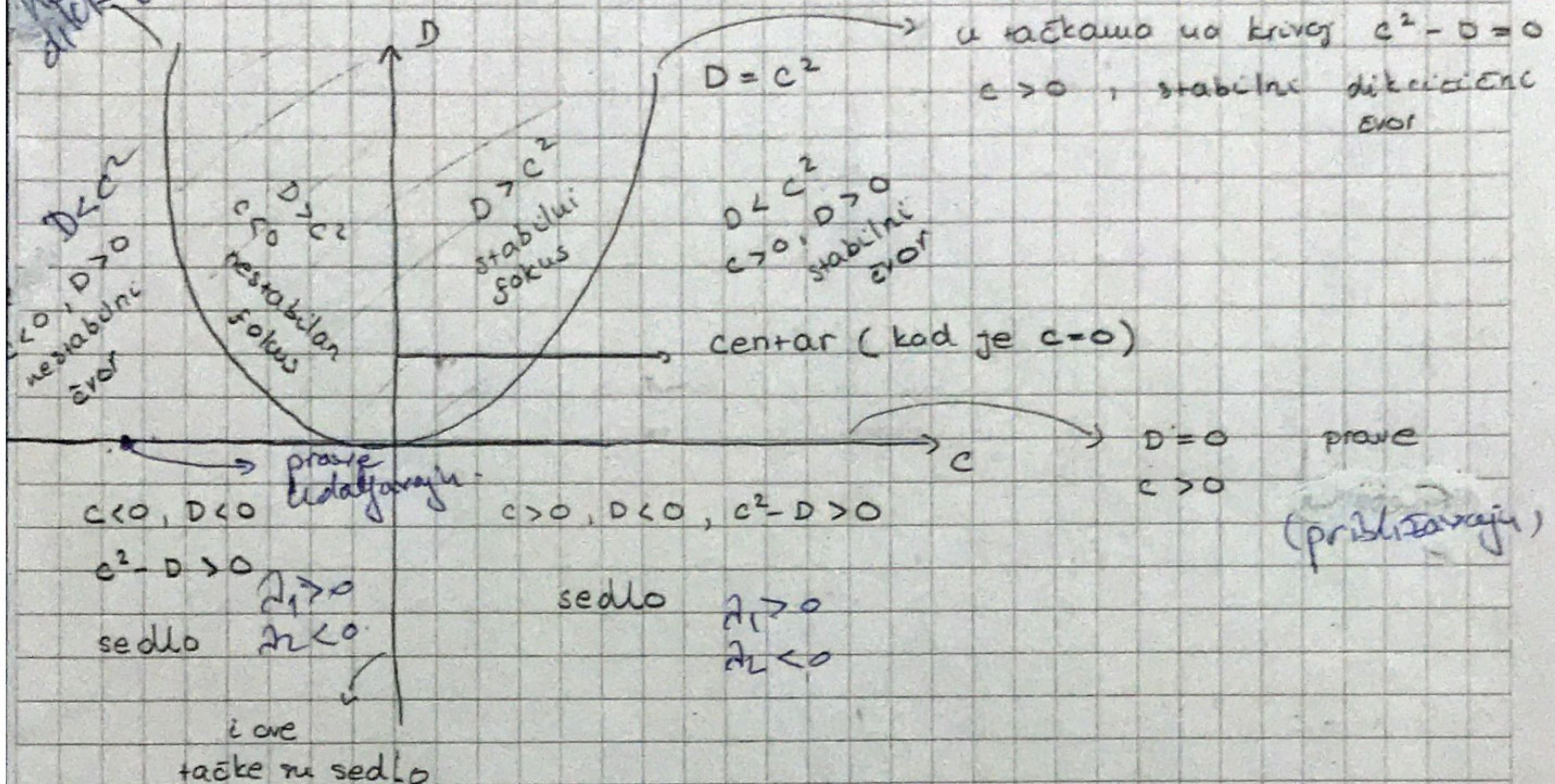
2° $a > 0 \Rightarrow$ nestabilni fokus

3° $a < 0 \Rightarrow$ stabilni fokus

2. $x_1 = -10x_2 + 10$

$$x_1' = x_1 - 2c x_2$$

$$\begin{array}{l} \text{det}(A - \lambda E) = \\ \text{det} \begin{vmatrix} -\lambda & -D \\ 1 & -2c - \lambda \end{vmatrix} = 0 \Rightarrow \lambda_{1,2} = -c \pm \sqrt{c^2 - D} \end{array}$$



③ $x_1' = x_1 + 3x_2$

$$\underline{x_2}^1 = -6x_1 - 5x_2$$

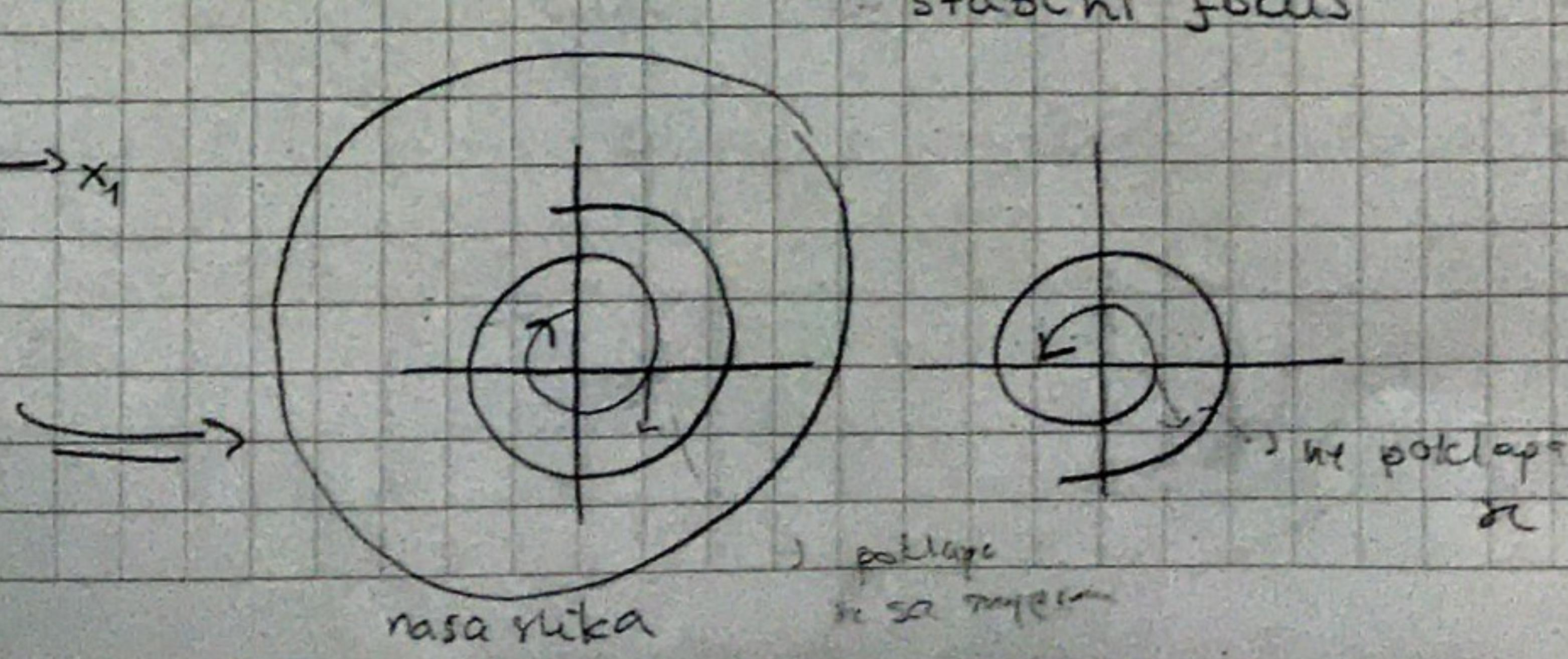
$$\text{A} = \begin{pmatrix} 1 & 3 \\ -6 & -5 \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{vmatrix} 1-\lambda & 3 \\ -6 & -5-\lambda \end{vmatrix} = (1-\lambda)(-5-\lambda) + 18 = \lambda^2 + 4\lambda + 13 = 0$$

(o, i) - proizvoljno

$$x_1 = 3 \quad \uparrow$$

$$x_2 = -5$$



$$4. \quad x' = -2x - 5y$$

$$y' = 2x + 2y$$

2

$$\det(A - \lambda E) = \dots$$

$$\lambda_{1,2} = \pm i\sqrt{6} \text{ center}$$

$$x^2 + y^2 \text{ max, min}$$

← Traciamo ose

$$y = kx$$

$$\text{ultimo su: } x' = -2x - 5y$$

$$y' = 2x + 2y$$

$$2x \cdot x' + 2y \cdot y' = 0$$

$$2x(-2x - 5y) + 2y(2x + 2y) = 0$$

$$2x(-2x - 5kx) + 2kx(2x + 2kx) = 0$$

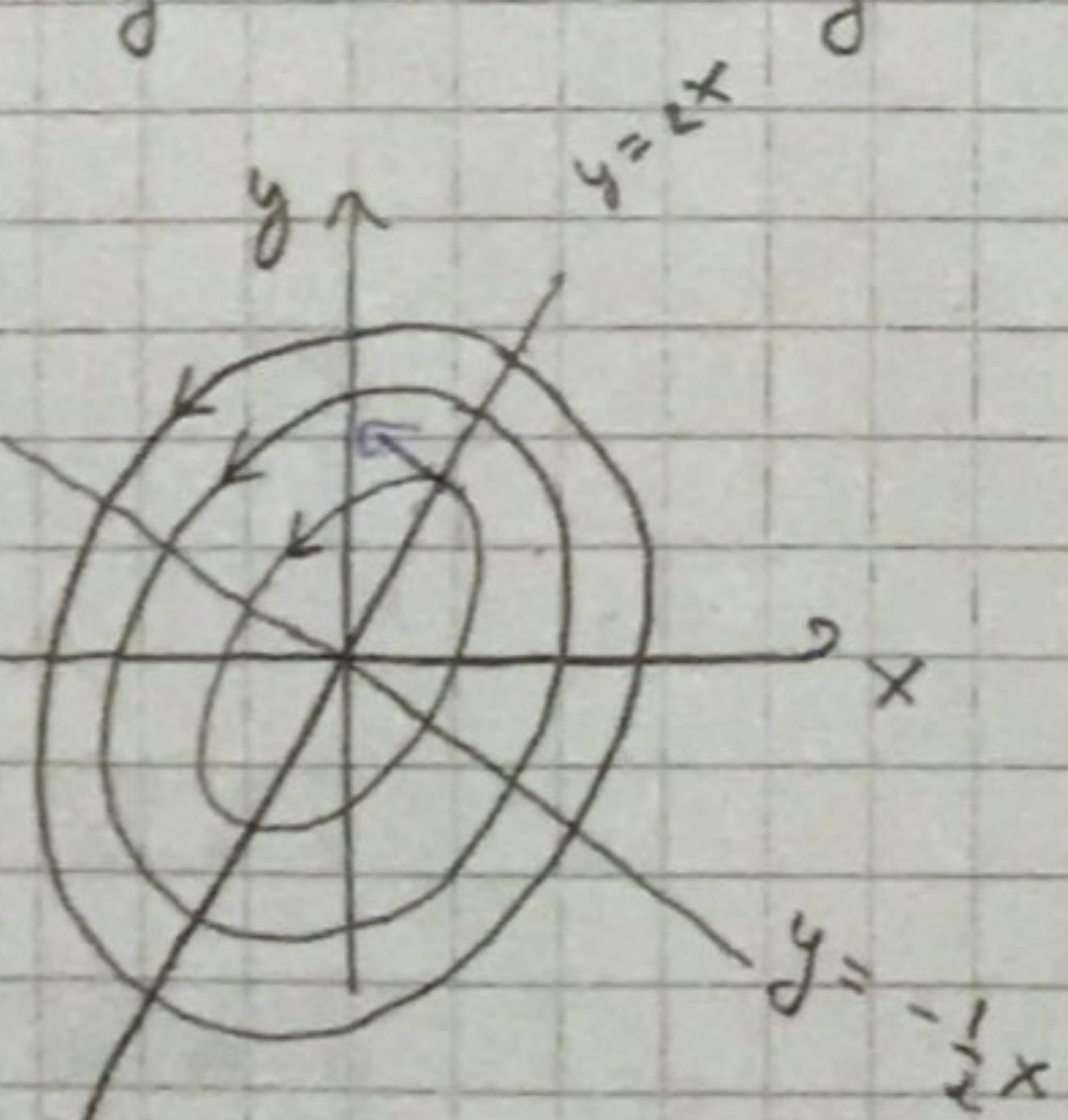
$$x^2(-4 - 10k + 4k^2) = 0$$

$$4k^2 - 6k - 4 = 0$$

$$2k^2 - 3k - 2 = 0$$

$$k = 2 \quad v \quad k = -\frac{1}{2}$$

$$y = 2x \quad y = -\frac{1}{2}x \quad \leftarrow \text{nase ose}$$

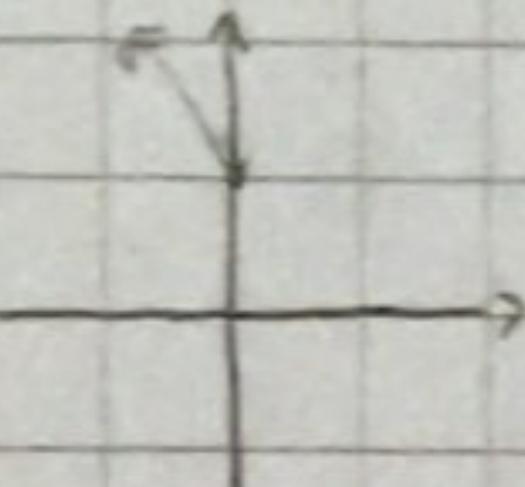


(1,2)

$$x' = -12 \quad \downarrow$$

$$y' = 6 \quad \uparrow$$

(rujer)



$$5) \begin{aligned} x' &= 3x - 4y \\ y' &= x - 2y \end{aligned}$$

$$\Rightarrow A = \begin{pmatrix} 3 & -4 \\ 1 & -2 \end{pmatrix}$$

$$\det(A - \lambda E) = \dots$$

$$\begin{aligned} \lambda_1 &= 2 \\ \lambda_2 &= -1 \end{aligned} \quad \left. \begin{array}{l} \text{sedlo} \\ \text{sa} \end{array} \right\}$$

I način

$$A h_1 = 2 h_1$$

$$(A - 2E) h_1 = 0$$

$$\begin{pmatrix} 1 & -4 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad h_{11} - 4h_{12} = 0$$

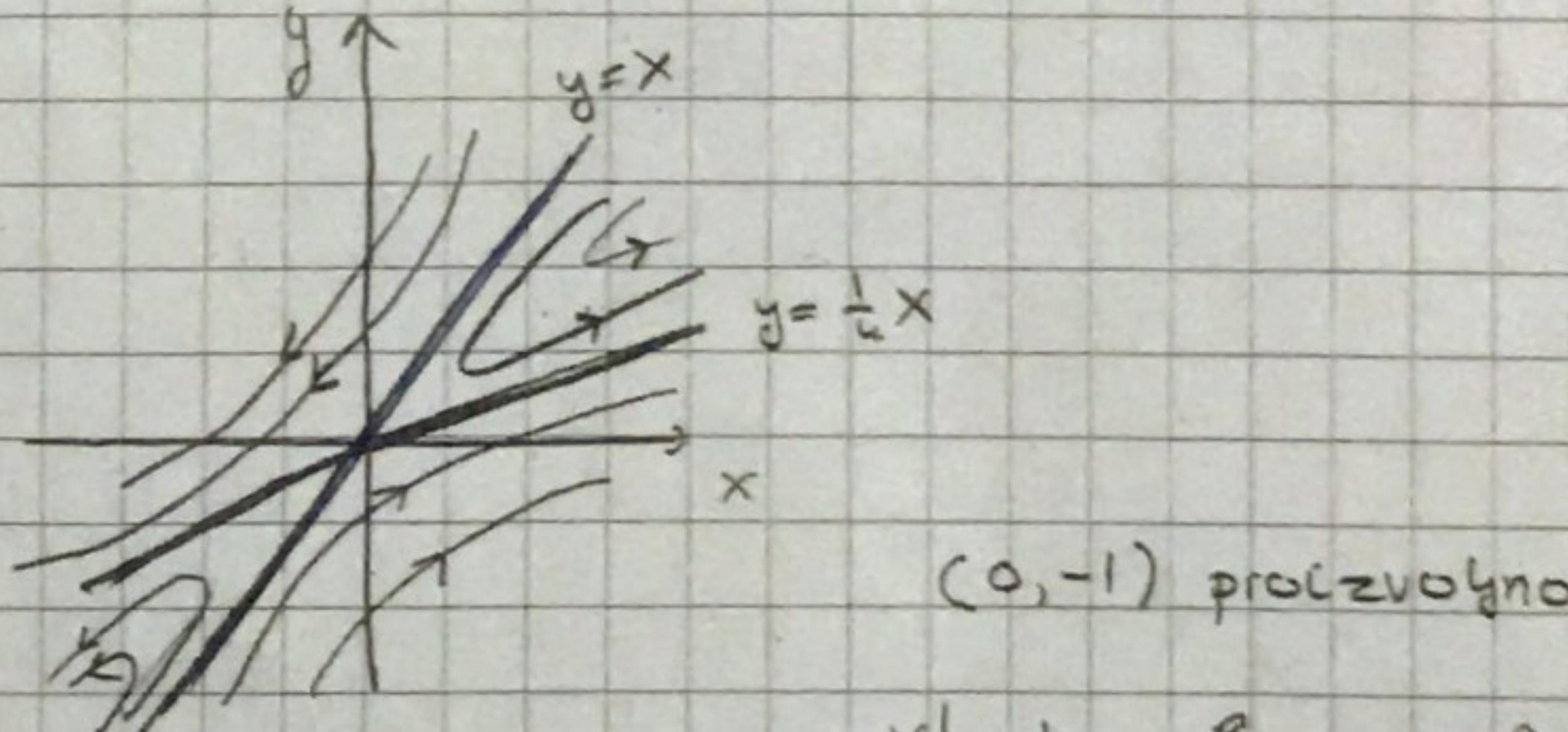
$$h_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$y = \frac{1}{4}x \quad \left. \begin{array}{l} \text{prava koja je vezana} \\ \text{za ovim vektorom} \end{array} \right\}$$

$$A h_2 = -1 h_2$$

$$h_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$y = x$$



$$(1, \frac{1}{2}) \rightarrow (1, 0) \rightarrow$$

$$(-1, -\frac{1}{2}) \rightarrow (-1, 0)$$

$$y' = 2 \quad \text{P}$$

$$(0, 1) \rightarrow (-4, -2) \quad \text{P}$$

II način

$$y = kx$$

$$\frac{dy}{dx} = \frac{x - 2y}{3x - 4y}$$

$$k = \frac{x - 2kx}{3x - 4kx}$$

$$k = \frac{1 - 2k}{3 - 4k} \Rightarrow 4k^2 - 5k + 1 = 0 \Rightarrow k = 1, k = \frac{1}{4}$$

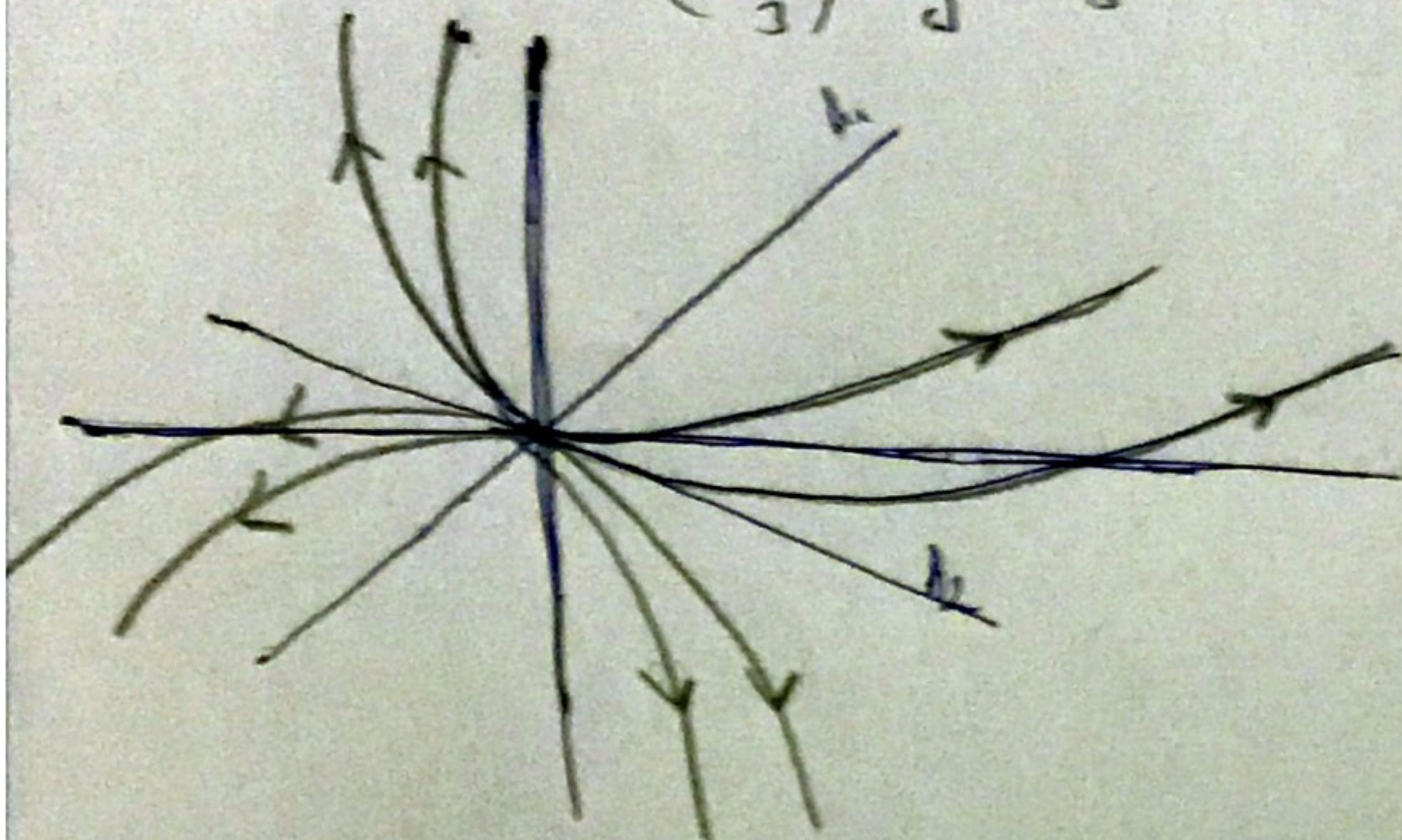
$$\textcircled{6} \quad \begin{aligned} x' &= 2x + 3y \\ y' &= x + 4y \end{aligned}$$

$$A = \dots$$

$$\lambda_1 = 5, \quad \lambda_2 = 1 \quad \text{neutrales Zentrum}$$

$$\lambda_1 = 5 \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad y = x$$

$$\lambda_2 = 1 \rightarrow \begin{pmatrix} 1 \\ -\frac{1}{3} \end{pmatrix} \quad y = -\frac{1}{3}x$$



$$\frac{dy}{dx} = \frac{x+4y}{2x+3y} \Rightarrow k = \dots$$

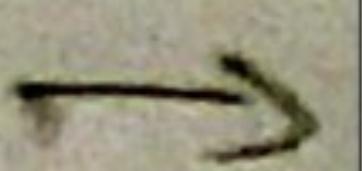
$$y = kx$$

$$\textcircled{7} \quad \begin{aligned} x' &= 3x+4y \\ y' &= y-x \end{aligned}$$

$$\lambda_1 = \lambda_2 = 2 \quad \text{neutrales Drehzentrum Zentrum}$$

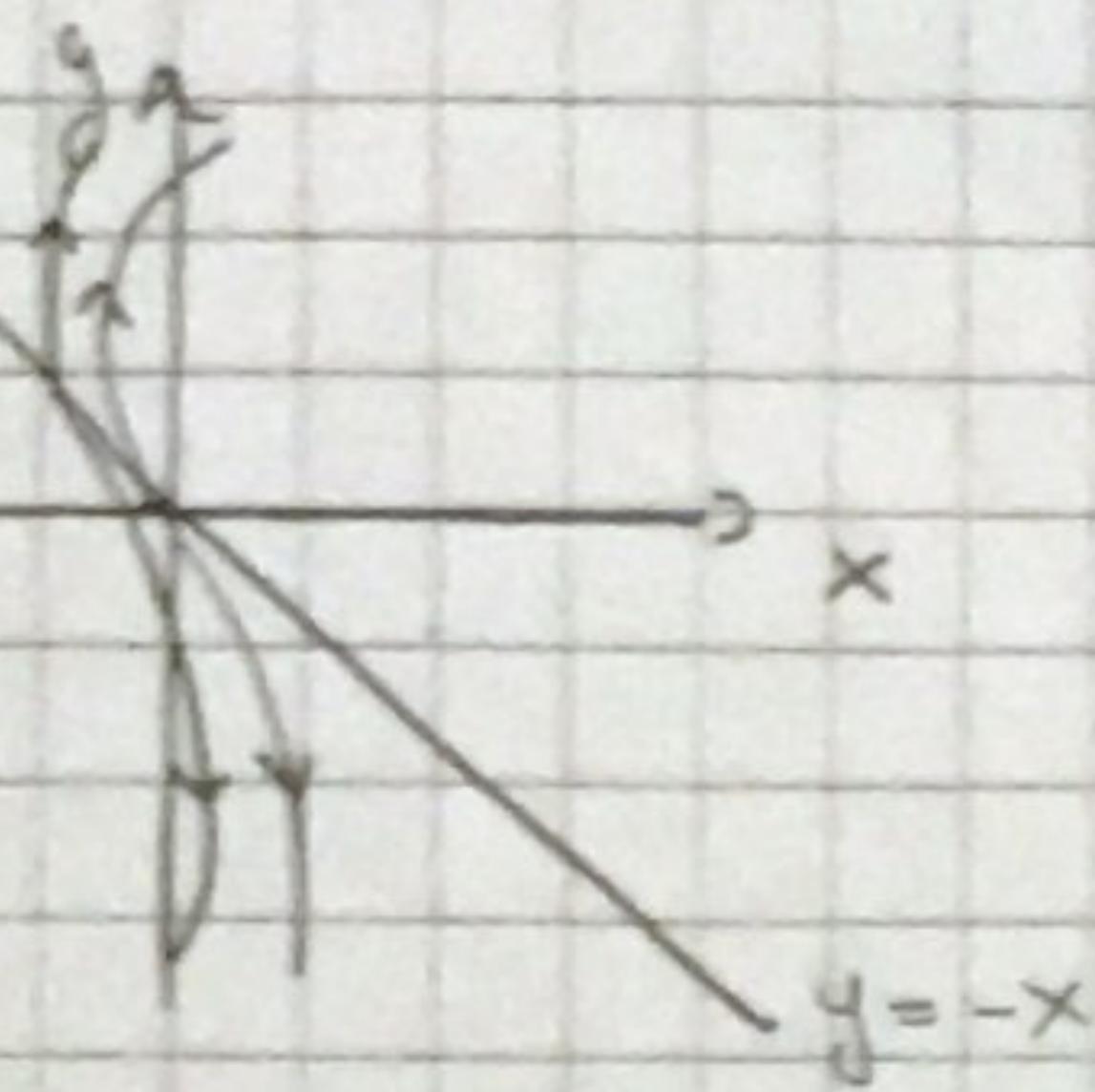
$$\text{rang}(A - 2E) = \text{rang} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = 1$$

2-1 = 1 neg. Vektor



$$(A - 2E) \begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$h_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad y = -x$$

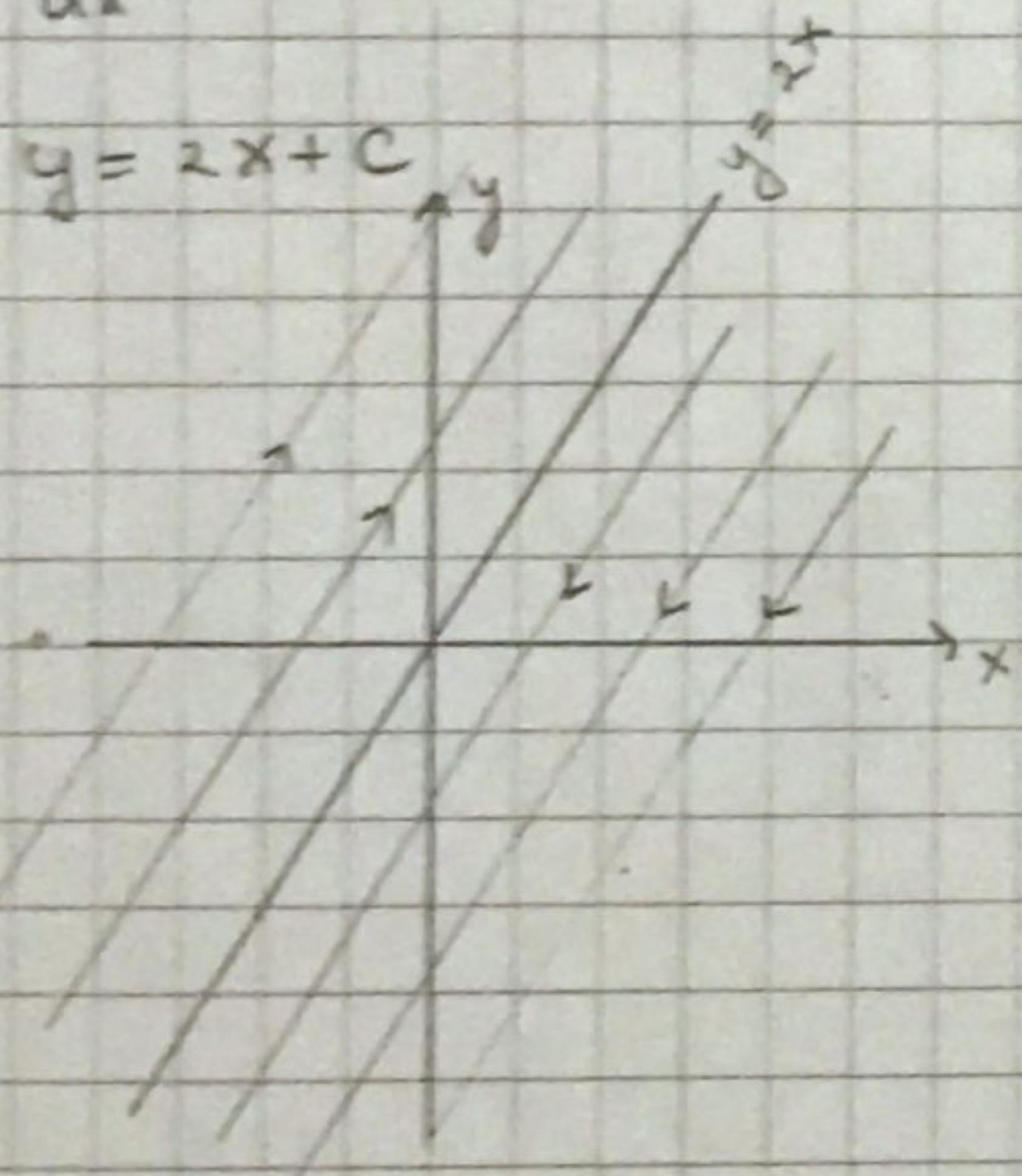


8. $x' = y - 2x$

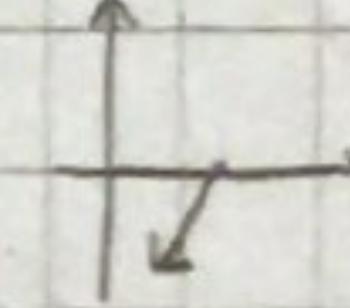
$y' = 2y - 4x$

$\Rightarrow \lambda_1 = \lambda_2 = 0$ prave (cijela fazna ravan)

$$\frac{dy_1}{dx} = 2$$

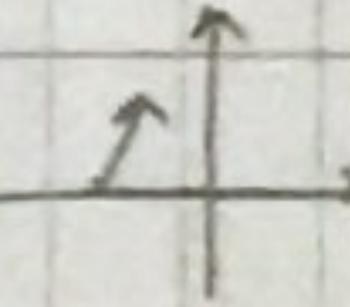


(1,0) pravouglno $x' = -2 \downarrow$



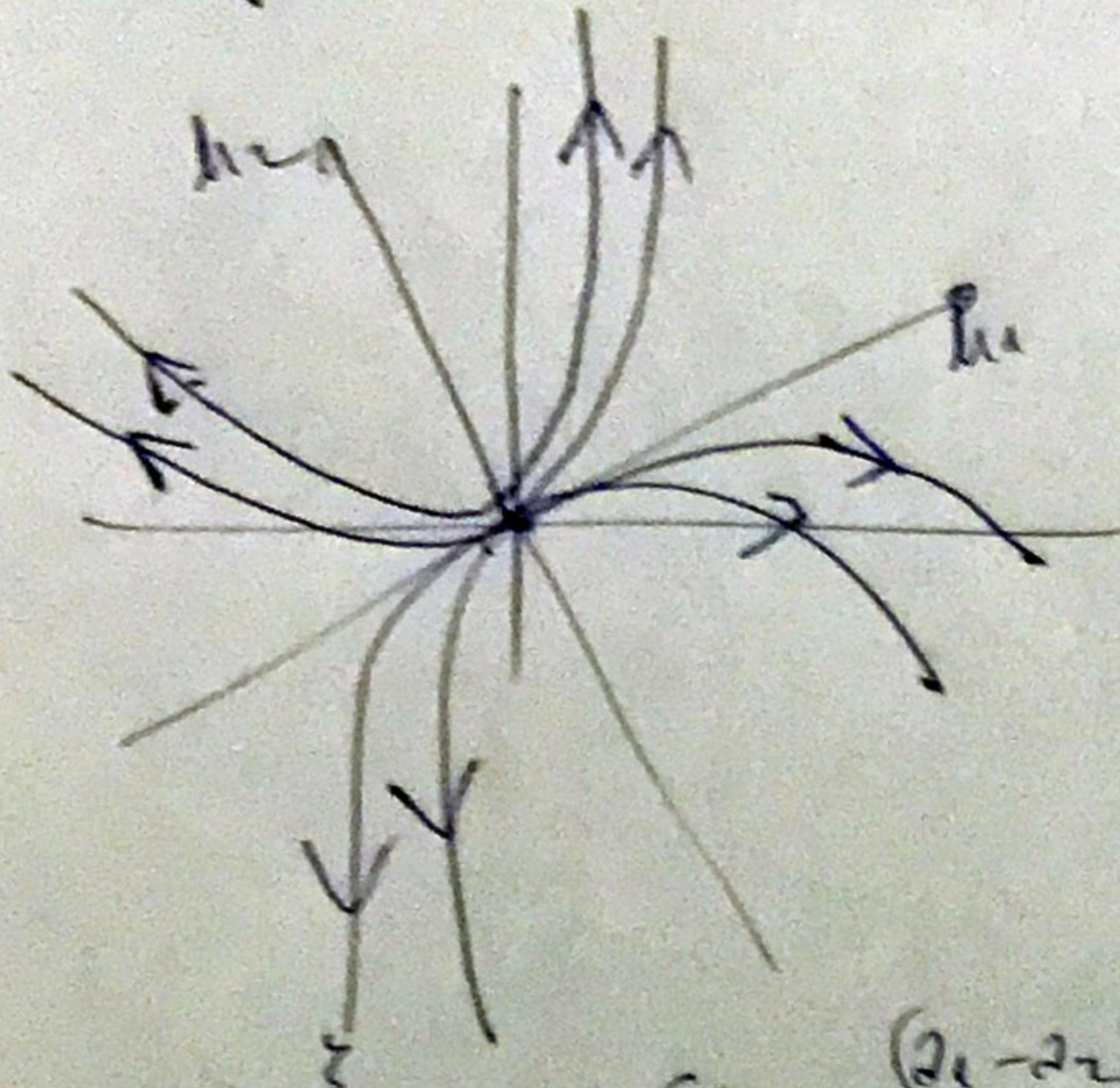
$y' = -4 \downarrow$

(-1,0) pravouglno $x' = 2 \uparrow$



$y' = 4 \uparrow$

$$\text{If } x = c_1 e^{d_1 t} + c_2 e^{d_2 t} \quad 0 < d_1 < d_2$$



$$x' = c_1 e^{d_1 t}$$

$$m = c_2 e^{d_2 t} \quad c_1, c_2 > 0.$$

$$\begin{aligned} x' &\rightarrow \infty, \quad t \rightarrow \infty \\ m &\rightarrow \infty, \quad t \rightarrow \infty \\ x' &\rightarrow 0, \quad t \rightarrow -\infty \\ m &\rightarrow 0, \quad t \rightarrow -\infty. \end{aligned}$$

$$\frac{x'}{m} = \frac{c_1}{c_2} e^{(d_1 - d_2)t} \xrightarrow[t \rightarrow \infty]{} 0$$

$$x = c_1 \overset{(x)}{+} c_2 e^{d_2 t} \cdot \overset{(x')}{m}$$

$$\begin{aligned} x' &= c_2 \\ m &= c_2 e^{d_2 t} \end{aligned}$$

$$d_1 < 0 < d_2$$

$$x = c_1 e^{d_1 t} + c_2 e^{d_2 t}$$

$$x' = c_1 e^{d_1 t} \xrightarrow[t \rightarrow \infty]{} 0$$

$$m = c_2 e^{d_2 t} \xrightarrow[t \rightarrow \infty]{} \infty$$